Spin models with random anisotropy and reflection symmetry

Pasquale Calabrese,[1,2,*] Andrea Pelissetto,[3,†] and Ettore Vicari[4,‡]
[1]*Scuola Normale Superiore and INFN, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*
[2]*Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, United Kingdom*
[3]*Dipartimento di Fisica dell'Università di Roma I and INFN, I-00185 Roma, Italy*
[4]*Dipartimento di Fisica dell'Università and INFN, Via Buonarroti 2, I-56127 Pisa, Italy*
(Received 3 December 2003; published 10 September 2004)

We study the critical behavior of a general class of cubic-symmetric spin systems in which disorder preserves the reflection symmetry $s_a \to -s_a$, $s_b \to s_b$ for $b \neq a$. This includes spin models in the presence of random cubic-symmetric anisotropy with probability distribution vanishing outside the lattice axes. Using nonperturbative arguments we show the existence of a stable fixed point corresponding to the random-exchange Ising universality class. The field-theoretical renormalization-group flow is investigated in the framework of a fixed-dimension expansion in powers of appropriate quartic couplings, computing the corresponding β functions to five loops. This analysis shows that the random Ising fixed point is the only stable fixed point that is accessible from the relevant parameter region. Therefore, if the system undergoes a continuous transition, it belongs to the random-exchange Ising universality class. The approach to the asymptotic critical behavior is controlled by scaling corrections with exponent $\Delta = -\alpha_r$, where $\alpha_r \simeq -0.05$ is the specific-heat exponent of the random-exchange Ising model.

DOI: 10.1103/PhysRevE.70.036104 PACS number(s): 64.60.Ak, 75.10.Nr, 75.10.Hk

I. INTRODUCTION AND SUMMARY

The critical behavior of magnetic systems in the presence of quenched disorder has been the subject of extensive theoretical and experimental study. An important class of systems is formed by amorphous alloys of rare earths with aspherical electron distribution and transition metals, for instance $TbFe_2$ and YFe_2. These systems are modeled [1,2] by the Heisenberg model with random uniaxial single-site anisotropy, or, in short, by the random-anisotropy model (RAM)

$$\mathcal{H}_{\rm RAM} = -J \sum_{\langle xy \rangle} \vec{s}_x \cdot \vec{s}_y - D_0 \sum_x (\vec{u}_x \cdot \vec{s}_x)^2, \tag{1}$$

where $\vec{s}_x$ is an M-component spin variable, $\vec{u}_x$ is a unit vector describing the local (spatially uncorrelated) random anisotropy, and D_0 is the anisotropy strength. In amorphous alloys the distribution of $\vec{u}_x$ is usually taken to be isotropic, since in the absence of crystalline order there is no preferred direction. On the other hand, in polycrystalline materials, for instance in the Laves-phase intermetallic $(Dy_xY_{1-x})Al_2$ compounds studied in Refs. [3,4], the distribution of $\vec{u}_x$ is expected to have only the lattice symmetry.

The critical behavior, and in particular the nature of the low-temperature phase, of a generic system with random anisotropy depends on the probability distribution of the random vector $\vec{u}_x$. In the isotropic case, i.e., when the probability distribution is uniformly weighted over the $(M-1)$-dimensional sphere, the Imry-Ma argument [5,6] forbids the appearance of a low-temperature phase with nonvanishing magnetization for $d<4$. This still allows the presence of a finite-temperature transition with a low-temperature phase in which correlation functions decay algebraically, as happens in the two-dimensional *XY* model. Such behavior has been predicted for the RAM with isotropic distribution in Ref. [7] and it has been recently supported by a $4-\epsilon$ study [8,9] using the functional renormalization group (RG) [10]. On the other hand, standard field-theoretical perturbative approaches do not find any evidence for a critical behavior with long-range correlations [11–14]. While experiments have not yet found evidence of low-temperature quasi-long-range order, numerical simulations seem to confirm the picture of Refs. [7–9], but are still contradictory as far as universality and behavior in the strong-anisotropy regime are concerned [15,16]. For these reasons, the critical behavior of the RAM can still be considered as an open problem.

The above arguments do not apply to spin models with the discrete anisotropic distribution introduced in Ref. [11], in which the vector $\vec{u}_x$ points only along one of the M lattice axes, i.e., it has the probability distribution

$$P_c(\vec{u}) = \frac{1}{2M} \sum_{a=1}^{M} [\delta^{(M)}(\vec{u} - \hat{x}_a) + \delta^{(M)}(\vec{u} + \hat{x}_a)], \tag{2}$$

where $\hat{x}_a$ is a unit vector that points in the positive a direction. This model, which we shall call the random cubic anisotropic model (RCAM), should have a standard order-disorder transition: The random discrete cubic anisotropy should stabilize a low-temperature phase with long-range ferromagnetic order. On the basis of two-loop calculations in field-theoretical frameworks, it has been argued [17,18] that the transition belongs to the universality class of the random-exchange Ising model (REIM) for any number M of components.

*Email address: calabres@df.unipi.it
†Email address: Andrea.Pelissetto@roma1.infn.it
‡Email address: vicari@df.unipi.it

In this paper we study the critical properties of the three-dimensional RCAM. We consider the field-theoretical approach based on the Landau-Ginzburg-Wilson φ^4 Hamiltonian [11]

$$\mathcal{H}_{\rm LGW} = \int d^d x \left\{ \sum_{i,a} \frac{1}{2}[(\partial_\mu \phi_{ai})^2 + r\phi_{ai}^2] + \frac{1}{4!}\sum_{ijab} (u_0 + v_0\delta_{ij} + w_0\delta_{ab} + y_0\delta_{ij}\delta_{ab})\phi_{ai}^2\phi_{bj}^2 \right\}, \quad (3)$$

where $a,b=1,\ldots,M$ and $i,j=1,\ldots,N$. In the limit $N\to 0$ the Hamiltonian (3) is expected to describe the critical behavior of the RCAM for M-component spins. Using nonperturbative arguments, we show that the field theory with Hamiltonian (3) has two stable fixed points (FP's). One of them belongs to the REIM universality class while the other corresponds to the O(N) model in the limit $N\to 0$, the so-called self-avoiding-walk universality class. Then we investigate the RG flow for the model with Hamiltonian (3) in the framework of a fixed-dimension expansion in powers of appropriate zero-momentum quartic couplings. We compute the corresponding Callan-Symanzik β functions to five loops. Their analysis shows that the only accessible stable FP from the region of parameters relevant for the RCAM is the REIM FP. This implies that the critical behavior of the RCAM (when the parameters allow a continuous transition) belongs to the REIM universality class, whose critical exponents are $\nu_r=0.683(3)$, $\alpha_r=-0.049(9)$, $\eta_r=0.035(2)$, etc. [19]. The approach to the REIM scaling behavior is characterized by very slowly decaying scaling corrections proportional to t^Δ with $\Delta=-\alpha_r\approx 0.05$, which is much smaller than the scaling-correction exponent of the REIM, which is $\Delta_r\approx 0.25$ [20,21]. Our results fully confirm and put on a firmer ground the conclusions of Refs. [17,18] based on two-loop perturbative calculations.

It is important to note that our results are specific to distributions that vanish everywhere outside the lattice axes, such as the one given in Eq. (2). Indeed, generic cubic-symmetric distributions $P(\vec{u})$, and in particular the isotropic one, give rise to an additional quartic term that should be added to the effective Hamiltonian (3), i.e,

$$z_0 \sum_{ijab} \phi_{ai}\phi_{bi}\phi_{aj}\phi_{bj}. \quad (4)$$

The REIM FP is unstable with respect to this perturbation. We shall evaluate the corresponding crossover exponent, finding $\phi_z=0.79(4)$. Therefore, even small differences from the discrete distribution $P_c(\vec{u})$ cause a crossover to a different critical behavior. Nonetheless, when $P_c(\vec{u})$ turns out to be a good effective approximation—this might be the case in some crystalline cubic-symmetric random-anisotropy systems—REIM critical behavior may be observed in a preasymptotic region.

The general Landau-Ginzburg-Wilson Hamiltonian (3) can also be recovered by considering systems with cubic anisotropy such that disorder preserves the symmetry $s_{x,a}\to -s_{x,a}$, $s_{x,b}\to s_{x,b}$, $b\neq a$. A general Hamiltonian with this property is given by

$$\mathcal{H} = -J\sum_{\langle xy\rangle} \vec{s}_x\cdot\vec{s}_y - K\sum_x\sum_a s_{x,a}^4 - D_0\sum_x\sum_a q_{x,a}s_{x,a}^2, \quad (5)$$

where $s_x^2=1$ and $\vec{q}_x$ is a random vector with a probability distribution that is invariant under the interchange $q_a \leftrightarrow q_b$. The exact reflection symmetry at fixed disorder—this symmetry is not present in generic models of type (1)—is the key property that allows the RCAM and the more general class of models (5) to have a standard order-disorder transition with a low-temperature magnetized phase.

The paper is organized as follows. In Sec. II we apply the replica method to the φ^4 theory corresponding to models (1) and (5), determining the corresponding ϕ^4 Hamiltonians that are the basis of the field-theoretical approach. In Sec. III we discuss some general properties of the theory (3). In particular, we discuss the crossover behavior when randomness is weak, and we prove that the REIM FP is stable by evaluating its stability eigenvalues. In Sec. IV we investigate the RG flow by computing and analyzing the five-loop fixed-dimension expansion of the β functions associated with the zero-momentum quartic couplings. In Appendix A we report a six-loop calculation of the RG dimensions of the bilinear operators in cubic-symmetric models that are used in the discussion of the stability of the FP's. Appendix B reports the proof of some identities used in the paper.

II. EFFECTIVE Φ^4 HAMILTONIANS

The mapping of the RAM Hamiltonian (1) to an effective translationally invariant ϕ^4 Hamiltonian was originally discussed in Ref. [11]. In order to replace fixed-length spins with unconstrained variables, one performs a Hubbard-Stratonovich transformation. Then, for the purpose of studying the critical behavior one considers the continuum limit of the resulting Hamiltonian and truncates its potential to fourth order. This leads to an effective continuum φ^4 Hamiltonian for an M-component real field φ_a,

$$\mathcal{H}_{\varphi^4} = \int d^d x \left[\frac{1}{2}(\partial_\mu\vec{\varphi})^2 + \frac{1}{2}r\vec{\varphi}^2 - D(\vec{u}\cdot\vec{\varphi})^2 + \frac{1}{4!}v_0(\vec{\varphi}^2)^2 \right], \quad (6)$$

where $\vec{u}_x$ is an external spatially uncorrelated vector field with parity-symmetric distribution $P(\vec{u})$ and D is proportional to D_0. We relax here the condition $\vec{u}_x^2=1$ and require only that $\langle \vec{u}_x^2\rangle=1$, thereby fixing the normalization of D. Using the standard replica trick it is possible to replace the quenched average with an annealed one. The system is replaced by N noninteracting copies with annealed disorder. Then, by integrating over disorder, one obtains the following effective Hamiltonian:

$$H_{\rm repl} = \int d^d x \left[\sum_{i,a}\frac{1}{2}(\partial_\mu\phi_{ai})^2 + \frac{1}{2}r\sum_{ia}\phi_{ai}^2 + \frac{1}{4!}v_0\sum_{ijab}\delta_{ij}\phi_{ai}^2\phi_{bj}^2 + R(\phi)\right], \quad (7)$$

where $a,b=1,\ldots,M$, $i,j=1,\ldots,N$, and

$$R(\phi) = -\ln \int d^N u P(\vec{u}) \exp\left(D \sum_{iab} u_a u_b \phi_{ai} \phi_{bi} \right). \tag{8}$$

In the limit $N \to 0$ the Hamiltonian (7) is equivalent to the Hamiltonian (6) with quenched disorder. The expansion in powers of the field ϕ can be expressed in terms of the moments of the distribution $P(\vec{u})$,

$$M_{a_1 a_2 \ldots a_k} \equiv \int d^N u P(\vec{u}) u_{a_1} u_{a_2} \ldots u_{a_k}. \tag{9}$$

One obtains

$$H_{\rm repl} = \int d^d x \left[\frac{1}{2} \sum_{ia} (\partial_\mu \phi_{a,i})^2 + \frac{1}{2} r \sum_{ia} \phi_{a,i}^2 + \frac{1}{4!} v_0 \sum_{iab} \phi_{ai}^2 \phi_{bi}^2 - D \sum_{iab} M_{ab} \phi_{ai} \phi_{bi} + \frac{1}{2} D^2 \left(\sum_{iab} M_{ab} \phi_{ai} \phi_{bi} \right)^2 - \frac{1}{2} D^2 \sum_{ijabcd} M_{abcd} \phi_{ai} \phi_{bi} \phi_{cj} \phi_{dj} + O(\phi^6) \right]. \tag{10}$$

Let us consider the case in which all field components become critical at T_c. This is achieved if the distribution $P(\vec{u})$ is such that

$$M_{ab} = \frac{1}{M} \delta_{ab}. \tag{11}$$

This condition is satisfied if $P(u)$ is cubic symmetric. Under this further assumption, the fourth moment M_{abcd} can be written as

$$M_{abcd} = A(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) + B \delta_{abcd}, \tag{12}$$

where A and B are parameters depending on the distribution; they satisfy the Cauchy inequalities $A(M+2)+B \geqslant 1/M$ and $3A+B \geqslant 1/M^2$. It follows that

$$H_{\rm repl} = \int d^d x \left[\frac{1}{2} \sum_{ia} (\partial_\mu \phi_{ai})^2 + \frac{1}{2}(r - D/M) \sum_{ia} \phi_{ai}^2 + \frac{1}{4!} v_0 \sum_{iab} \phi_{ai}^2 \phi_{bi}^2 + \frac{D^2}{2M^2}(1 - M^2 A) \left(\sum_{ia} \phi_{ai}^2 \right)^2 - AD^2 \sum_{ijab} \phi_{ai} \phi_{bi} \phi_{aj} \phi_{bj} - \frac{BD^2}{2} \sum_{ija} \phi_{ai}^2 \phi_{aj}^2 + O(\phi^6) \right]. \tag{13}$$

In conclusion, for generic cubic-symmetric distributions $P(\vec{u})$ the Hamiltonian that should describe the critical behavior of the corresponding RAM is

$$\mathcal{H} = \int d^d x \left\{ \frac{1}{2} \sum_{ia} (\partial_\mu \phi_{ai})^2 + \frac{1}{2} r \sum_{ia} \phi_{ai}^2 + \frac{1}{4!} \sum_{ijab} [(u_0 + v_0 \delta_{ij} + w_0 \delta_{ab} + y_0 \delta_{ij} \delta_{ab}) \phi_{ai}^2 \phi_{bj}^2 + z_0 \phi_{ai} \phi_{bi} \phi_{aj} \phi_{bj}] \right\}, \tag{14}$$

where the term proportional to y_0 has been added because it is generated by RG iterations whenever $w_0 \neq 0$. It should be noticed that such a term arises naturally if one considers that, if the system is only cubic symmetric, quartic single-ion terms must be included. In this case it is natural to consider

$$\mathcal{H} = -J \sum_{\langle xy \rangle} \vec{s}_x \cdot \vec{s}_y - D_0 \sum_x (\vec{u}_x \cdot \vec{s}_x)^2 + K \sum_x \sum_a s_{x,a}^4, \tag{15}$$

and the corresponding φ^4 Hamiltonian

$$\mathcal{H} = \int d^d x \left[\frac{1}{2} (\partial_\mu \vec{\varphi})^2 + \frac{1}{2} r \vec{\varphi}^2 - D(\vec{u} \cdot \vec{\varphi})^2 + \frac{1}{4!} v_0 (\vec{\varphi}^2)^2 + \frac{1}{4!} y_0 \sum_a \varphi_a^4 \right]. \tag{16}$$

The Hamiltonian (14) was originally introduced in Ref. [12] to describe magnetic systems with single-ion anisotropy and nonmagnetic impurities.

There are two interesting particular cases. First, one may consider an O(M)-invariant pure system coupled to an isotropic distribution $P(u)$. In this case $K=0$ in Eq. (15)—therefore, $y_0=0$—and $B=0$ in Eq. (12), so that $w_0=0$. These conditions are preserved under renormalization by the presence of the O(M) invariance. Note that this is not the case if $K \neq 0$, i.e., if $y_0 \neq 0$. Distributions such that $B=0$ (these distributions are not necessarily isotropic) give apparently $w_0 = 0$; however, such a condition is not preserved under renormalization if $z_0 \neq 0$.

A second interesting case corresponds to distributions $P(u)$ such that $A=0$ in Eq. (12). It is easy to show that distributions $P(u)$ with this property are simple generalizations of the distribution (2). Explicitly, they have the form

$$P(u) = \frac{1}{M} \sum_a f(u_a) \prod_{b \neq a} \delta(u_b), \tag{17}$$

where $f(x)$ is a normalized probability distribution with unit variance. If $A=0$, Eq. (13) implies $z_0=0$. Such a condition is stable under renormalization. Indeed, the transformation $\phi_{ai} \to -\phi_{ai}$ for fixed a and i is a symmetry of the Hamiltonian with $z_0=0$, but not of the term proportional to z_0. This symmetry is due to the fact that, for distributions of type (17), we can write $(\vec{s} \cdot \vec{u})^2 = \Sigma_a u_a^2 s_a^2$, which is symmetric under the transformations $s_a \to -s_a$ at fixed u. In other words, the theory with $z_0=0$ describes models in which the reflection symmetry of the spins is also preserved at fixed disorder.

In the case of discrete cubic-symmetric distributions of type (17), we have

$$u_0 = \frac{12 D^2}{M^2}, \quad w_0 = -12 B D^2. \tag{18}$$

These conditions imply

$$u_0 > 0, \quad w_0 < 0, \quad M u_0 + w_0 \leqslant 0, \tag{19}$$

where the last condition follows from the bound $B \geqslant 1/M$. The equality $Mu_0 + w_0 = 0$ is obtained by using the distribution (2). The relations (18) and (19) should be considered as indicative, since the mapping between the φ^4 Hamiltonian (16) and the general Hamiltonian (14) also gives rise to higher-order terms.

It is also interesting to consider the effective continuum Hamiltonian corresponding to Eq. (5). In this case we obtain

$$\mathcal{H}_{\varphi^4} = \int d^d x \left[\frac{1}{2}(\partial_\mu \vec{\varphi})^2 + \frac{1}{2} r \vec{\varphi}^2 - D \sum_a q_a \varphi_a^2 + \frac{1}{4!} v_0 (\vec{\varphi}^2)^2 + \frac{1}{4!} y_0 \sum_a \varphi_a^4 \right]. \quad (20)$$

If $P(q)$ is invariant under the interchanges $q_a \leftrightarrow q_b$, we can write for the first moments $M_a = a$ and $M_{ab} = b + c\delta_{ab}$. A simple calculation gives again the general Hamiltonian (14) with $z_0 = 0$ and

$$u_0 = \frac{D^2}{2}(a^2 - b), \; w_0 = -\frac{cD^2}{2}. \quad (21)$$

Since $b + c \geqslant a^2$, we obtain

$$u_0 + w_0 \leqslant 0. \quad (22)$$

Equality is obtained for $P(q) = \Pi_a \delta(q_a - 1)$ (in this case however $w_0 = 0$). Finally, note that if $c = 0$ then we have $w_0 = 0$. Such a condition is stable under renormalization, and thus this class of models is expected to have a different critical behavior. It corresponds to the one of the randomly dilute cubic models discussed in Ref. [22]. Distributions with this property are, however, quite peculiar. They have the general form

$$P(q) = f(q_1) \prod_{a=2}^{M} \delta(q_1 - q_a). \quad (23)$$

The stability region of the quartic potential in the φ^4 Hamiltonian (3) is given by the conditions

$$Nu_0 + v_0 + Nw_0 + y_0 > 0, \quad (24)$$

$$NMu_0 + Mv_0 + Nw_0 + y_0 > 0, \quad (25)$$

$$u_0 + v_0 + w_0 + y_0 > 0, \quad (26)$$

$$Mu_0 + Mv_0 + w_0 + y_0 > 0. \quad (27)$$

However, as discussed in Ref. [12], in the zero-replica limit $N \to 0$, the only relevant stability conditions are those obtained by using replica-symmetric configurations. Therefore, for the RCAM one should consider only Eqs. (24) and (25) with $N = 0$, i.e.,

$$v_0 + y_0 > 0 \text{ if } v_0 > 0,$$

$$Mv_0 + y_0 > 0 \text{ if } v_0 < 0. \quad (28)$$

Equivalently, the relevant stability conditions can be obtained by considering the Hamiltonians (16) and/or (20).

III. GENERAL RENORMALIZATION-GROUP PROPERTIES

A. Fixed points of the theory

The properties of the RG flow are essentially determined by its FP's. Most of them can be identified by considering the theories obtained when some of the quartic parameters vanish. For example, we can easily recognize (a) the $O(M \times N)$ theory for $v_0 = w_0 = y_0 = 0$; (b) N decoupled $O(M)$ theories for $u_0 = w_0 = y_0 = 0$; (c) M decoupled $O(N)$ theories for $u_0 = v_0 = y_0 = 0$; (d) $M \times N$ decoupled Ising theories for $u_0 = v_0 = w_0 = 0$; (e) the MN model (see, e.g., Refs. [23,24]) for $w_0 = y_0 = 0$; (f) the NM model for $v_0 = y_0 = 0$; (h) N decoupled M-component cubic models for $u_0 = w_0 = 0$; (i) M decoupled N-component cubic models for $u_0 = v_0 = 0$; (j) the $(M \times N)$-component cubic model for $v_0 = w_0 = 0$; (k) the randomly diluted M-component cubic model (see Ref. [22]) for $w_0 = 0$ and $N = 0$; and (l) the tetragonal model [23,25] for $M = 2$ and $w_0 = 0$.

The FP's of these theories are also FP's of the enlarged model (3). Of course, there may also be FP's that are not related to the above particular cases. Their presence can be investigated by low-order ϵ-expansion calculations. First-order ϵ-expansion calculations [11] show the presence of 14 FP's for $M \neq 2$ and of 13 FP's for $M = 2$. As in the REIM case, at two-loop order other $O(\sqrt{\epsilon})$ FP's appear [12]: four FP's for $M \neq 2$ and six FP's for $M = 2$ [17]. The two-loop ϵ-expansion results are summarized in Refs. [17,18]. In Table I we report the leading ϵ-expansion terms for the location of the FP's (in the minimal-subtraction renormalization scheme) and the corresponding stability eigenvalues. The only stable FP's are the O(0) and the REIM FP's, which are already present in models (a) and (i), respectively. These results have also been supported by two-loop fixed-dimension calculations [17]. In order to understand the relevance of the various FP's for the RCAM, we need to check which one is accessible from the region of quartic parameters relevant for the three-dimensional RCAM. This issue will be investigated in Sec. IV by computing and analyzing five-loop series in the framework of the fixed-dimension expansion.

B. Crossover behavior close to the pure spin model

The $O(M)$-symmetric FP located on the v axis describes the critical properties of the pure spin system in the absence of cubic anisotropy. It is interesting to compute the crossover exponent in the presence of random anisotropy. Setting $t_p \equiv (T - T_p)/T_p$, where $T_p \equiv T_c(D_0 = 0)$ is the critical temperature in the absence of anisotropy, in the limit $t_p \to 0$ and $D_0 \to 0$ the singular part of the free energy can be written as

$$\mathcal{F} = |u_t|^{2-\alpha} f(D_0^2 |u_t|^{-\phi_D}), \quad (29)$$

where $u_t \approx t_p + a_1 D_0^2 + a_2 t_p^2 + \cdots$ is the scaling field associated with temperature, α is the specific-heat exponent in the $O(M)$ theory, ϕ_D is the crossover exponent, and $f(x)$ is a scaling function. As a consequence of Eq. (29), for sufficiently small D_0 the critical-temperature shift is given by

$$\Delta T_c(D_0) \equiv T_c(D_0) - T_c(0) \approx a D_0^{2/\phi_D} + b D_0^2 + c D_0^4 + \cdots. \quad (30)$$

The crossover exponent ϕ_D is related to the largest positive RG dimension of the perturbations at the $O(M)$ FP that are present in the Hamiltonian (3), i.e., of the terms proportional to u_0, w_0, and y_0. For $u_0 = w_0 = y_0 = 0$, the Hamiltonian (3)

TABLE I. Fixed points of the Hamiltonian (3) near four dimensions. We report the leading nontrivial contribution of the expansion in powers of ϵ, taken from Refs. [11,17]. Here, $K_d=(4\pi)^d\Gamma(d/2)/2$, $\alpha_\pm=(M-4\pm\sqrt{M^2+48})/8$, $\beta_\pm=-(M+12\pm\sqrt{M^2+48})/6$, $A_{\pm\pm}=6\alpha_\pm+3\beta_\pm+M+6$. The general expressions for the stability eigenvalues of FP's XI–XIV are rather cumbersome. We report their numerical values only for $M=3$.

		v^*/K_d	u^*/K_d	w^*/K_d	y^*/K_d	Stability eigenvalues
I	Gaussian	0	0	0	0	$\omega_u=\omega_v=\omega_w=\omega_y=-\epsilon$
II	O(M)	$\frac{6}{M+8}\epsilon$	0	0	0	$\omega_v=\epsilon, \omega_u=-\frac{4-M}{M+8}\epsilon, \omega_w=-\frac{4+M}{M+8}\epsilon, \omega_y=\frac{4-M}{M+8}\epsilon$
III	O(0)	0	$\frac{3}{4}\epsilon$	0	0	$\omega_u=\epsilon, \omega_v=\omega_w=\omega_y=\epsilon/2$
IV	O(0)	0	0	$\frac{3}{4}\epsilon$	0	$\omega_u=\omega_v=-\epsilon/2, \omega_w=\epsilon, \omega_y=\epsilon/2$
V	Ising	0	0	0	$\frac{2}{3}\epsilon$	$\omega_u=\omega_v=\omega_w=-\epsilon/3, \omega_y=\epsilon$
VI		$\frac{3}{2(M-1)}\epsilon$	$\frac{3(M-4)}{8(M-1)}\epsilon$	0	0	$\omega_1=\epsilon, \omega_2=\omega_y=\frac{4-M}{4(M-1)}\epsilon, \omega_w=-\frac{4+M}{4(M-1)}\epsilon$
VII		0	$\frac{3}{2}\epsilon$	$-\frac{3}{2}\epsilon$	0	$\omega_1=\omega_v=\epsilon, \omega_3=\omega_y=-\epsilon$
VIII	Cubic	$\frac{2}{M}\epsilon$	0	0	$\frac{2(M-4)}{3M}\epsilon$	$\omega_u=\omega_2=-\frac{4-M}{3M}\epsilon, \omega_w=-\frac{4+M}{3M}\epsilon, \omega_4=\epsilon$
IX	$M\neq 2$	$\frac{1}{M-2}\epsilon$	$\frac{M-4}{4(M-2)}\epsilon$	0	$\frac{M-4}{3(M-2)}\epsilon$	$\omega_1=\epsilon, \omega_2=\frac{4-M}{6(M-2)}\epsilon, \omega_w=\frac{-(4+M)}{6(M-2)}, \omega_4=\frac{-(4-M)}{6(M-2)}\epsilon$
X		0	$\frac{1}{2}\epsilon$	$-\frac{1}{2}\epsilon$	$\frac{2}{3}\epsilon$	$\omega_1=\epsilon, \omega_v=\omega_3=\epsilon/3, \omega_4=-\epsilon/3$
XI		$\frac{3}{A_{++}}\epsilon$	$\frac{3\alpha_+}{A_{++}}\epsilon$	$\frac{3(M+4)}{4A_{++}}\epsilon$	$\frac{3\beta_+}{A_{++}}\epsilon$	$\omega_1=\epsilon, \omega_2=1.33\epsilon, \omega_3=\omega_4=-1.43\epsilon$ (for $M=3$)
XII		$\frac{3}{A_{+-}}\epsilon$	$\frac{3\alpha_+}{A_{+-}}\epsilon$	$\frac{3(M+4)}{4A_{+-}}\epsilon$	$\frac{3\beta_-}{A_{+-}}\epsilon$	$\omega_1=\epsilon, \omega_2=-\omega_3=0.371\epsilon, \omega_4=-0.344\epsilon$ (for $M=3$)
XIII		$\frac{3}{A_{-+}}\epsilon$	$\frac{3\alpha_-}{A_{-+}}\epsilon$	$\frac{3(M+4)}{4A_{-+}}\epsilon$	$\frac{3\beta_+}{A_{-+}}\epsilon$	$\omega_1=\epsilon, \omega_2=-\omega_3=0.435, \omega_4=-0.403\epsilon$ (for $M=3$)
XIV		$\frac{3}{A_{--}}\epsilon$	$\frac{3\alpha_-}{A_{--}}\epsilon$	$\frac{3(M+4)}{4A_{--}}\epsilon$	$\frac{3\beta_-}{A_{--}}\epsilon$	$\omega_1=\epsilon, \omega_2=\omega_3=-3.32\epsilon, \omega_4=-3.08\epsilon$ (for $M=3$)
XV	REIM	0	0	$\mp\sqrt{\frac{54}{53}}\sqrt{\epsilon}$	$\pm\frac{4}{3}\sqrt{\frac{54}{53}}\sqrt{\epsilon}$	$\omega_u=\omega_v=\omega_1=\pm\sqrt{\frac{24}{53}}\sqrt{\epsilon}, \omega_2=2\epsilon$
XVI	REIM	0	$\mp\sqrt{\frac{54}{53}}\sqrt{\epsilon}$	0	$\pm\frac{4}{3}\sqrt{\frac{54}{53}}\sqrt{\epsilon}$	$\omega_w=\omega_v=-\omega_1=\mp\sqrt{\frac{24}{53}}\sqrt{\epsilon}, \omega_2=2\epsilon$
XVII	$M=2$	$\mp 2\sqrt{\frac{54}{53}}\sqrt{\epsilon}$	$\pm\sqrt{\frac{54}{53}}\sqrt{\epsilon}$	0	$\pm\frac{4}{3}\sqrt{\frac{54}{53}}\sqrt{\epsilon}$	$\omega_w=\omega_3=-\omega_1=\mp\sqrt{\frac{24}{53}}\sqrt{\epsilon}, \omega_2=2\epsilon$

describes N decoupled O(M)-symmetric systems. The RG dimension of the terms proportional to u_0 and w_0 can be determined by writing [11]

$$\sum_{abij}(u_0+w_0\delta_{ab})\phi_{ai}^2\phi_{bj}^2 = M(Mu_0+w_0)\sum_{ij}\mathcal{E}_i\mathcal{E}_j + w_0\sum_{ija}\mathcal{T}_{aai}\mathcal{T}_{aaj}, \tag{31}$$

where

$$\mathcal{E}_i \equiv \frac{1}{M}\sum_a \phi_{ai}^2, \; \mathcal{T}_{abi} \equiv \phi_{ai}\phi_{bi} - \delta_{ab}\mathcal{E}_i. \tag{32}$$

The bilinears $\mathcal{E}_i$ and $\mathcal{T}_{abi}$ are, respectively, the energies and the quadratic spin-2 operators of the N decoupled models. If $y_E=1/\nu$ and y_T are the corresponding RG dimensions, the two terms given above have RG dimensions $y_u=2y_E-3=\alpha/\nu$ and $y_w=2y_T-3$. The perturbation proportional to y_0 does not couple the different replicas and therefore its RG dimension y_y is simply the RG dimension of the cubic perturbation, which is related to the RG dimension of the spin-4 perturbation of the O(M) FP [26,27]. Therefore, the O(M) FP is perturbed by three terms of RG dimensions y_u, y_w, and y_y, which can be determined using known results for the RG dimensions of generic operators in an O(M) theory; see, e.g., Refs. [28,25] for reviews of results. Since α is negative for $M\geqslant 2$, we have $y_u<0$ and therefore the corresponding term is always irrelevant. The exponent y_T has been obtained by using field-theoretical [26] and Monte Carlo methods [29]: field-theoretical analyses give $y_T=1.766(6)$ for $M=2$ and $y_T=1.790(3)$ for $M=3$, while Monte Carlo simulations give $y_T=1.756(2)$ for $M=2$ and $y_T=1.787(2)$ for $M=3$. Correspondingly, we find $y_w=0.532(12)$ and $y_w=0.511(6)$ for $M=2$, and $y_w=0.580(6)$ and $y_w=0.573(3)$ for $M=3$. Therefore, the perturbation proportional to w_0 is always relevant. Finally, using the results of Refs. [26,27] for the spin-4 perturbations at the O(M) FP, we have $y_y=-0.103(8)$ for $M=2$ and $y_y=0.013(6)$ for $M=3$. This implies that the y term is irrelevant for $M=2$, but relevant for $M=3$. In conclusion, for both $M=2$ and $M=3$, the most relevant quartic perturbation is given by the w term, which determines the crossover from the pure critical behavior in the limit of small anisotropy strength. Therefore, $\phi_D=y_w\nu=0.357(3)$ for $M=2$ and $\phi_D=y_w\nu=0.412(3)$ for $M=3$. In the crossover limit in which $D_0^2|u_t|^{-\phi_D}$ is held fixed, the operators with RG dimensions y_u and y_y give rise to scaling corrections. In particular, there are corrections proportional to t^{Δ_y}, with $\Delta_y\equiv y_y\nu-\phi_D$, $\Delta_y=0.426(6)$ for $M=2$ and $\Delta_y=0.403(5)$ for $M=3$, which are more important than the usual O(M)-invariant corrections, which vanish as t^Δ, with $\Delta\approx 0.54$ for $M=2$ and $\Delta\approx 0.56$ for $M=3$ [25].

It is worth mentioning that the scaling behavior (29) with the same crossover exponent ϕ_D also holds for a RAM with generic distribution $P(\vec{u})$, and in particular for the isotropic case. Indeed, the additional term proportional to z_0 appearing in the Hamiltonian (14) has the same RG dimension of the w

term at the O(M) FP. This can be inferred by rewriting

$$\sum_{abij} \phi_{ai}\phi_{bi}\phi_{aj}\phi_{bj} = \sum_{abij} \mathcal{T}_{abi}\mathcal{T}_{abj} + M\sum_{ij} \mathcal{E}_i\mathcal{E}_j, \qquad (33)$$

where $\mathcal{T}_{abi}$ and $\mathcal{E}_i$ are defined in Eq. (32). The first term is the most relevant one and therefore we obtain $y_z=2y_T-3$ and also $y_z=y_w$.

Let us note that the relatively small value of ϕ_D makes the measurement of ϕ_D from the critical-temperature shift for small random anisotropy rather difficult. Indeed, in Eq. (30) the term D_0^{2/ϕ_D} is suppressed with respect to the first two analytic terms proportional to D_0^2 and D_0^4, since $2/\phi_D \approx 4.9$ ($2/\phi_D \approx 5.6$) for $M=3$ ($M=2$). This explains the results of Ref. [4] which measured T_c in crystalline Laves-phase $(Dy_xY_{1-x})Al_2$ for different values of x. Since $D_0 \to 0$ as $x \to 1$ [30], they were able to measure $\Delta T_c(D_0)$ for $D_0 \to 0$. The experimental results were fitted assuming $\Delta T_c(D_0) \sim D_0^{2/\psi}$, obtaining $\psi=0.80(8)$.This result is in substantial agreement with the theoretical prediction $\Delta T_c(D_0) \sim D_0^2$, but does not provide information on the crossover exponent ϕ_D.

For $M \geqslant 3$, pure systems with the Hamiltonian (15) do not have a critical behavior in the O(M) universality class; see, e.g., Refs. [23,27,22]. If the system has [111] as the easy direction, its critical behavior belongs to a different universality class with reduced cubic symmetry, while systems with [110] easy axis are expected to show a first-order transition. In the latter type of system, randomness may soften the first-order transition. This issue will be discussed in Sec. IV C. On the other hand, we now show that in cubic systems with [111] easy axis randomness is a relevant perturbation and therefore, for small randomness, these systems show a crossover behavior with positive exponent ϕ_D; cf. Eq. (29). For $u_0=w_0=0$ the Hamiltonian (3) reduces to the one for N decoupled systems with cubic symmetry. The RG dimensions of the terms proportional to u_0 and w_0 at the cubic FP provide the crossover exponent ϕ_D. In order to determine them, we use again Eq. (31). The RG dimension of $\mathcal{E}_i$ is $y_E=1/\nu$, where ν is the correlation-length exponent, while that of $\mathcal{U}_{ai} \equiv \mathcal{T}_{aai}$, y_U, is computed in Appendix A by resumming its six-loop perturbative expansion. Thus, the RG dimensions y_u and y_w of the two terms appearing in the right-hand side of Eq. (31) are given by $y_u=2y_E-3=\alpha/\nu$ and $y_w=2y_U-3$, respectively. Since $\alpha<0$ at the cubic FP for any $M \geqslant 3$, the first term is irrelevant. On the other hand, the estimates of y_U reported in Appendix A show that $y_w>0$ for any $M \geqslant 3$. For example $y_w=0.549(14)$ for $M=3$, and therefore $\phi_D =0.387(14)$.

Note that in a generic cubic-symmetric RAM, one should also consider perturbations proportional to z_0. We use again Eq. (33). However, in the presence of cubic symmetry T_{abi} [cf. Eq (32)] is not an irreducible tensor. One must consider separately $\mathcal{U}_{ai} \equiv \mathcal{T}_{aai}$ and $\mathcal{T}_{abi}$ with $a \neq b$, which have different RG dimensions y_U and y_T. Therefore, the term proportional to z_0 is the sum of three terms of RG dimensions $2y_E-3=\alpha/\nu$, $2y_U-3$, and $2y_T-3$. The last one is the largest, so that $y_z=2y_T-3$. Using the results of Appendix A for $M=3$, we find $y_z=0.600(4)$. The exponent y_z is larger than y_w. Therefore, a generic cubic-symmetric RAM shows a different crossover behavior with crossover exponent $\phi_D=y_z\nu =0.427(3)$.

C. Stable fixed points

The critical behavior in the presence of random anisotropy should be described by the stable FP of the theory (3) which is accessible from the RCAM. The two-loop ϵ-expansion calculations of Ref. [17] summarized in Sec. III A find two stable FP's. One of them is located on the u axis, and it is associated with the O(0) or self-avoiding-walk universality class. This FP is also stable in three dimensions. Indeed, the terms proportional to v_0, w_0, and y_0 are interactions transforming as the spin-4 representation of the O($M \times N$) group. Therefore, they have the same RG dimension which is given by $y_{v,w,y}=-0.37(5)$, obtained in Ref. [26] from the analysis of six-loop fixed-dimension and five-loop ϵ series. It was argued in Ref. [17], on the basis of two-loop calculations, that the O(0) FP is not accessible from the parameter region relevant for the RCAM. This will be confirmed by the five-loop analysis of the RG flow presented in Sec. IV.

For $u_0=v_0=0$ the Hamiltonian (3) corresponds to a cubic-symmetric model and, for $N \to 0$, it is the sum of M independent models that are the field-theoretical analog of the REIM. We will now show that the REIM FP is stable in the theory (3). It is sufficient to show that the terms proportional to u_0 and v_0 are irrelevant. For this purpose, we rewrite

$$\sum_{ijab} (u_0 + v_0\delta_{ij})\phi_{ai}^2\phi_{bj}^2 = N(Nu_0 + v_0)\sum_a \mathcal{E}_a^2 + v_0\sum_{ai} \mathcal{U}_{ai}^2, \qquad (34)$$

where $\mathcal{E}_a=(1/N)\Sigma_i\phi_{ai}^2$ and $\mathcal{U}_{ai}=\phi_{ai}^2-\mathcal{E}_a$. For $N \to 0$, $\mathcal{E}_i$ and $\mathcal{U}_{ai}$ have the same RG dimension (see Appendix A for the proof), $y_E=y_U=1/\nu_r$, where ν_r is the correlation-length critical exponent of the REIM universality class. Therefore, the RG dimension of the perturbation is given by $y_{uv}=2y_E-3 =\alpha_r/\nu_r$, where α_r is the REIM specific-heat exponent. Since α_r is negative (see the estimates reported in Refs. [25,31,19]), the REIM FP is stable. Using the recent Monte Carlo results reported in Ref. [19], we finally arrive at the estimate $y_{uv} \approx -0.07$. As we shall see in Sec. IV, the REIM FP turns out to be accessible to the RCAM, and no other stable FP exists in the region relevant for the RCAM. Therefore, the REIM universality class describes the critical properties of the RCAM in the case it undergoes a continuous transition. Estimates of several universal quantities for the REIM universality class can be found in Refs. [25,31,19,32]. Note, however, that the critical exponent controlling the leading scaling corrections differs from the one for the REIM, which is $\Delta_r \approx 0.25$ [20,21]. In the RCAM the leading scaling correction is due to the Hamiltonian terms proportional to u_0 and v_0. They cause slowly decaying corrections of order t^{Δ} with

$$\Delta = -\alpha_r = 0.049(9). \qquad (35)$$

If $P(q)=\Pi_a P_a(q_a)$, i.e., the probability distributions of the variables q_a are independent, the stability of the REIM FP can also be proved by starting directly from Eq. (20). Indeed, such a Hamiltonian corresponds to M random-exchange φ^4 models coupled by the term proportional to v_0. Such a term has the form $\sum_{ab}\mathcal{E}_a\mathcal{E}_b$, where $\mathcal{E}_a=(1/N)\varphi_a^2$ is the energy of the REIM. Therefore, this perturbation has RG dimension $2/\nu_r-3=\alpha_r/\nu_r$, which is negative. Thus, the coupling among the models is irrelevant, and thus it does not change the universality class of the system. If $P(q)$ does not factorize, the M φ^4 models are also coupled by disorder. The above-reported analysis shows that also this coupling is irrelevant, its RG dimension being $\alpha_r/\nu_r<0$.

As discussed in Sec. II, in the case of a generic random cubic-symmetric distribution $P(\vec{u})$, the Hamiltonian (14) also contains the term proportional to z_0. It is important to note that the REIM FP is unstable with respect to this perturbation, since its RG dimension y_z is positive at the REIM FP. The dimension of this perturbation, y_z, can be computed by rewriting the term proportional to z_0 as

$$\sum_{abij}\phi_{ai}\phi_{bi}\phi_{aj}\phi_{bj}=\sum_{ab}\sum_{i\neq j}\mathcal{T}_{aij}\mathcal{T}_{bij}+\sum_{ab}\sum_{i}\mathcal{U}_{ai}\mathcal{U}_{bi}+N\sum_{ab}\mathcal{E}_a\mathcal{E}_b, \tag{36}$$

where $\mathcal{T}_{aij}\equiv\phi_{ai}\phi_{aj}$ with $i\neq j$. Therefore, this perturbation is the sum of three terms that have RG dimensions $2y_T-3$, $2y_U-3$, and $2y_E-3$. Using the results reported in Appendix A, one finds that the most relevant term is the first one, so that

$$y_z=2y_T-3. \tag{37}$$

Using the estimate $y_T=2.08(3)$ reported in Appendix A, one obtains

$$y_z=1.16(6),\ \phi_z\equiv y_z\nu=0.79(4), \tag{38}$$

where ϕ_z is the corresponding crossover exponent.

D. Critical behavior for infinitely strong random anisotropy

In this section, we wish to investigate the general model (5) in the limit of infinite disorder, showing that, under some mild hypotheses for the probability distribution $P(q)$, one has REIM critical behavior for $M=2$ and $M=3$ and no transition for $M\geqslant 4$. This analysis further confirms the results of Sec. III C.

We first consider the case $D_0\to+\infty$. We suppose that the distribution $P(q)$ is such that there is only one direction k such that $q_k=\max_b q_b$ (or at least that this condition is satisfied with probability 1). This is the case if the distribution is continuous and is also true for the distribution $P(q)$ derived from Eq. (2) (note that $q_a=u_a^2$). Because of the assumption on $P(q)$, for $D_0\to+\infty$ the spin $\vec{s}$ is constrained to lie along the k direction, i.e., $s_k=\pm1$, $s_a=0$ for $a\neq k$. Thus, in this limit we can rewrite the Hamiltonian in the following way. At each site we define M Ising variables $\sigma_{x,a}$ and M disorder variables $\rho_{x,a}$. The Ising variables assume values ±1, while the disorder variables assume values 0 and 1 with probabilities induced by the distribution of $\vec{q}$:

$$\rho_{x,a}=\begin{cases}1 & \text{if}\quad q_{x,a}>q_{x,b}\ \text{for every}\ b\neq a,\\ 0 & \text{otherwise}.\end{cases}$$

Then, the average value of a quantity $\mathcal{O}(s_{x,a})$ is given by

$$\overline{\langle O\rangle}=[\langle\mathcal{O}(\rho_{x,a}\sigma_{x,a})\rangle_\sigma]_\rho, \tag{39}$$

where $[\cdot]_\rho$ indicates the average over the disorder variables $\rho_{x,a}$ and $\langle\cdot\rangle_\sigma$ indicates the sample average with Hamiltonian

$$\mathcal{H}=-J\sum_a\sum_{\langle xy\rangle}\sigma_{x,a}\sigma_{y,a}\rho_{x,a}\rho_{y,a}. \tag{40}$$

If $\mathcal{O}$ depends only on a single component of the spins, say it depends only on $s_{x,1}$, we can integrate out $\sigma_{x,a}$ and $\rho_{x,a}$ for $a\geqslant 2$. Thus, the Hamiltonian becomes a REIM Hamiltonian with disorder $\rho_{x,1}$. Now, we use the symmetry of $P(q)$ to conclude that the probability that ρ_a is 1 must be independent of a. Since $\sum_a\rho_a$ is always equal to 1, we obtain that $\rho_{x,1}=1$ with probability $1/M$ and $\rho_{x,1}=0$ with probability $1-1/M$. Therefore, we obtain that correlation functions of s_1 are exactly equal to the correlation functions of the site-diluted Ising model with vacancy density $1-1/M$. Note that this result is not true for correlation functions that involve different components of the spins. Indeed, the Hamiltonian (40) corresponds to M REIMs, but they are coupled by the disorder variables. Thus, these correlation functions are not simply obtained by multiplying REIM correlation functions. These considerations allow us to predict the behavior of the model (5) for $D_0\to+\infty$. Since the REIM has a continuous transition for spin density $p>p_c$, $p_c=0.311\,608\,1(13)$ on a cubic lattice [33], we predict that the model has a continuous transition for $M=2$ and $M=3$ and no transition at all for $M\geqslant 4$.

Let us now consider the opposite case $D_0\to-\infty$. If the distribution $P(q)$ is such that there is only one direction k such that $q_k=\min_b q_b$, the previous argument applies with no changes. Note that the distribution (2) does not satisfy this condition for $M\geqslant 3$. Indeed, in the limit $D_0\to-\infty$ the spins are constrained to be orthogonal to $\vec{q}$, and therefore cannot be considered as Ising variables. In this particular case, the behavior at the transition, if it exists, is not predicted by this argument.

IV. RENORMALIZATION-GROUP FLOW IN THE QUARTIC-COUPLING SPACE

A. The fixed-dimension five-loop expansion

In this section we study the RG flow of the theory (3), determining the stable FP's and their attraction domain. For this purpose, we determine the five-loop perturbative expansion of the β functions in terms of appropriately defined zero-momentum quartic couplings at fixed dimension. In the present case we define u, v, w, and y by writing

$$\Gamma^{(4)}_{aibjckdl}(0)=mZ_\phi^{-2}\frac{16\pi}{3}(u\,R_{MN}A_{aibjckdl}+v\,R_MB_{aibjckdl}+w\,R_NC_{aibjckdl}+y\,D_{aibjckdl}), \tag{41}$$

where $R_K=9/(8+K)$, and A, B, C, and D are appropriate tensors defined so that at the tree level $u_0 = mu\, R_{MN}$, $v_0 = mv\, R_M$, $w_0 = mw\, R_N$, and $y_0 = my$. The mass m and the renormalization constant Z_ϕ are defined by

$$\Gamma^{(2)}_{aibj}(p) = \delta_{ab}\delta_{ij} Z_\phi^{-1}[m^2 + p^2 + O(p^4)]. \tag{42}$$

Here $\Gamma^{(4)}$ and $\Gamma^{(2)}$ are the four- and two-point one-particle irreducible correlation functions.

We computed the β functions to five loops. This required the calculation of 161 Feynman diagrams. We employed a symbolic manipulation program, which generated the diagrams and computed the symmetry and group factors of each of them. We used the numerical results compiled in Ref. [34] for the integrals associated with each diagram. The five-loop series of the β functions β_u, β_v, β_w, and β_y for the physically interesting cases $N=0$ and $M=2,3$ are reported in Ref. [35], with a discussion of their checks. At two loops the series agree with the expansions reported in Ref. [17]. The series for generic values of N and M are available on request.

Perturbative series are divergent and thus a careful analysis is needed in order to obtain quantitative predictions. In the case of systems without randomness, they are conjectured to be Borel summable and this allows one to use the Padé-Borel method or methods based on a conformal mapping [36]. In random systems, the perturbative approach faces additional difficulties: the perturbative series are expected not to be Borel summable [37,38]. Nonetheless, in the REIM case quite reasonable estimates of the critical exponents have been obtained by using the fixed-dimension expansion in $d=3$ (see, e.g., Refs. [25,31]). Similarly, the usual resummation methods applied to the RCAM expansions give quite stable results, at least when the quartic couplings are not too large, giving us confidence on the correctness of the conclusions.

B. The RG trajectories

The knowledge of the β functions allows us to study the RG flow in the space of the quartic renormalized couplings u, v, w, and y. For this purpose we follow closely Ref. [21]. The RG trajectories are lines starting from the Gaussian FP (located at $u=v=w=y=0$) along which the quartic Hamiltonian parameters u_0, v_0, w_0, and y_0 are kept fixed. The RG curves in the coupling space depend on three independent ratios of the quartic couplings. The RG trajectories can be determined by solving the differential equations

$$-\lambda \frac{dg_i}{d\lambda} = \beta_{g_i}, \tag{43}$$

where $g_i = u, v, w, y$, and $\lambda \in (0,\infty)$, with the initial conditions

$$g_i(0) = 0, \quad \left.\frac{dg_i}{d\lambda}\right|_{\lambda=0} = s_i, \tag{44}$$

where $s_1 = s_u \equiv u_0/|v_0|$, $s_3 = s_w \equiv w_0/|v_0|$, $s_4 = s_y \equiv y_0/|v_0|$, and $s_2 = +1$ if $v_0 > 0$, $s_2 = -1$ if $v_0 < 0$. The functions $g_i(\lambda, s_i)$ provide the RG trajectories in the renormalized-coupling space. The attraction domain of a FP g_i^* is given by the values of u_0, v_0, w_0, and y_0 corresponding to trajectories ending at g_i^*, i.e. trajectories for which $g_i(\lambda=\infty, s_i) = g_i^*$. We recall that the O($M$) FP is located on the v axis at $v^* \approx 1.40, 1.40$ for $M=2,3$ respectively [39,40]; the O(0) FP lies in the u axis at $u^* \approx 1.39$ (Refs. [39,40]); the REIM FP lies in the w-y plane at $w^* \approx -0.7$ and $y^* \approx 2.3$ (we report here the field-theoretical estimates of Ref. [24]; Monte Carlo estimates are given in Ref. [19]).

C. Results

In this section we report our analyses of the five-loop perturbative series. We have resummed the β functions by using the Padé-Borel method. The major numerical problem we faced was the fact that most of the approximants were defective in some region of the coupling space, forbidding a complete study of the RG flow. This is not unexpected since the perturbative series are not Borel summable. Approximant $[3/1]$ for β_u, $[4/1]$ for β_v, and $[3/2]$ for β_w with $b=1$ [41] were not defective in all the region of the RG flow we considered (in some cases the Padé approximant to the Borel transform had a pole on the positive real axis but far from the origin, in a region that gives a negligible contribution to the resummed function). On the other hand, all approximants for β_y were defective somewhere in the region we wished to investigate. For β_y we used approximant $[3/2]$ with $b=1$, that had the least extended defective region. All results we present here were obtained by using these approximants. It must be stressed that other choices gave results that were similar in the regions in which they were well defined.

First, we checked the general results reported in Sec. III. We considered the O(M), cubic, O(0), and REIM FP's and for each of them we determined the stability eigenvalues. The results are in full agreement with the conclusions of Sec. III, confirming that the O(0) and the REIM FP's are stable. Then, we looked for additional FP's in addition to those identified by the ϵ-expansion analysis of Sec. III A. For this purpose we considered the RG flow starting from arbitrary values of u, v, w, y. We only observed runaway trajectories or a flow toward either the REIM or the O(0) FP's, confirming that the REIM and the O(0) are the only stable FP's. In particular, trajectories corresponding to Hamiltonian parameters $w_0 < 0$, $u_0 > 0$, $u_0 + w_0 < 0$, and that satisfy the stability bound (28) never flow toward the O(0) FP, which is therefore not accessible from this region. They either flow toward the REIM FP or apparently run away toward infinity.

For the purpose of illustration, we first consider the case $y_0=0$, $w_0/u_0 = -M$, and $v_0 > 0$, which apparently corresponds to the model (6) with distribution (2) [cf. Eq. (18)]; note that $B=1/M$ in this case. In Fig. 1 we show the RG trajectories for $M=3$ for several values of $s_u > 0$. The approximant of β_y is defective for $0.05 \lesssim s_u \lesssim 0.3$ and y close to 1. This explains the sudden change of direction of the trajectory with $s_u = 0.3$ in Fig. 1 when y is close to 1. For $M=3$ ($M=2$) the RG trajectories appear to approach the REIM FP for $0 < s_u \lesssim 0.9$ ($0 < s_u \lesssim 1.4$). For larger values of s_u the flow runs close to regions in which some approximant is defective. The trajectories appear to flow toward infinity, but this could be an artifact of the resummation. In any case, if true, this would imply that the corresponding systems do not undergo

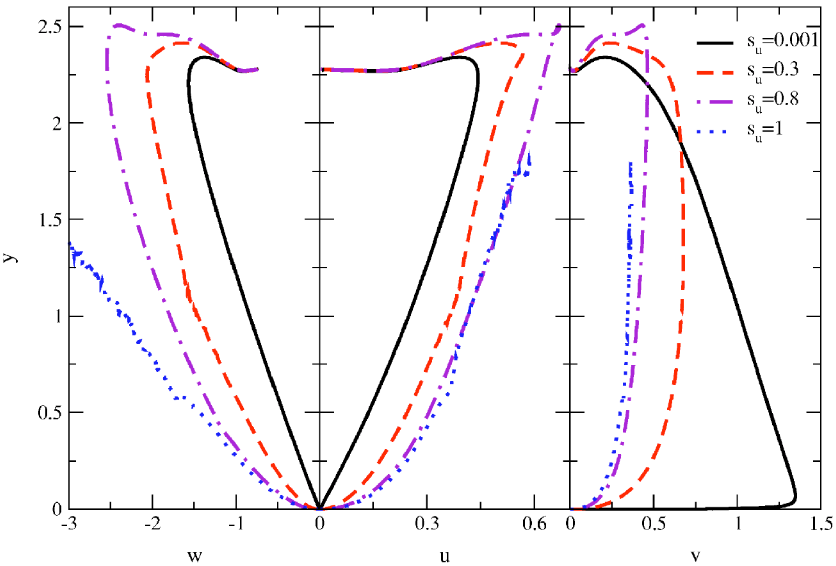

FIG. 1. Projections of the RG flow for the three-component case, $M=3$, in the y-w, y-v, and y-u planes, as a function of $s_u \equiv u_0/v_0$, for $u_0>0$, $v_0>0$, and $w_0<0$. Here $y_0=0$ and $3u_0+w_0=0$. The REIM FP corresponds to $u^*=v^*=0$, $w^*\approx -0.7$, and $y^*\approx 2.3$ (Ref. [24]).

a continuous transition. As a consequence, since s_u is directly related to the anisotropy strength D, the continuous transition would be expected to disappear for sufficiently large values of D. These conclusions do not immediately apply to fixed-length spin systems, i.e., to the Hamiltonian (1) since in this case $v_0=+\infty$ [42]. Thus, it is not clear which is the correct value of s_u even for $u_0=\infty$. The critical behavior of this system for strong disorder has been discussed in Sec. III D.

The qualitative picture does not change if we do not require $w_0/u_0=-M$ and $s_y=0$. For instance, we can consider the case $w_0/u_0=-M$, $v_0>0$, and arbitrary s_y. We are able to resum reliably the perturbative series for $s_y>-0.7$ and there we observe that some trajectories flow toward the REIM FP, while others run away to infinity. The attraction domain of the REIM FP enlarges with increasing s_y: it is approximately bounded by $s_u \lesssim 0.9+0.6s_y$ for $M=3$ ($s_u \lesssim 1.4+1.4s_y$ for $M=2$) in the region $-0.6\lesssim s_y \lesssim 0.3$ ($-0.7\lesssim s_y\lesssim 1$). For larger value of s_y, the attraction domain becomes even larger and extends beyond the lines reported above. For $s_y\lesssim -0.6$ some approximants become defective and we cannot reliably determine the RG flow. As in the case $s_y=0$, there is some evidence that trajectories flow toward infinity for $s_y\lesssim -1$, while for $-1\lesssim s_y \lesssim -0.6$ they may still flow to the REIM FP.

Then we investigated the behavior for $v_0<0$, although this region does not appear to be of physical interest. As in the pure case, all trajectories apparently run away to infinity.

Finally, let us discuss whether O(0) critical behavior can be observed by appropriately tuning the model parameters. We have investigated this question in detail. We find that the O(0) FP can be reached only if $u_0+w_0>0$, irrespective of the other parameters as long as $u_0>0$ and $w_0<0$. This result can be proved straightforwardly in the limiting case $v_0=0$. Indeed, since $\beta_v=0$ for $v=0$, if we start with $v_0=0$ the flow will be confined in the hyperplane $v=0$. But for $v=0$ we can use the identities

$$\beta_u(u,0,w,y) + \beta_w(u,0,w,y) = \beta_{\mathrm{REIM},u}(u+w,y),$$

$$\beta_y(u,0,w,y) = \beta_{\mathrm{REIM},y}(u+w,y), \qquad (45)$$

holding for $N=0$, where $\beta_{\mathrm{REIM},u}(u,y)$ and $\beta_{\mathrm{REIM},y}(u,y)$ are the β functions of the REIM model obtained by setting $v=w=0$ and $M=1$ (the corresponding six-loop series are reported in Ref. [27]). These identities are proved in Appendix B. They show that the flow for the couplings $u+w$ and y is identical to the flow observed in the random-exchange φ^4 theory. Therefore, for $u_0+w_0=0$ we observe pure Ising behavior, while for $u_0+w_0<0$ ($u_0+w_0>0$) RG trajectories flow toward the REIM [O(0)] FP. Therefore, if $w_0/u_0<-1$, as implied by Eq. (22), only REIM critical behavior can be observed. For $v_0>0$, similar conclusions are obtained numerically: The attraction domain of the O(0) FP is included in the region $w_0/u_0>-c$, where the constant c is positive and smaller than 1, depends on s_w, and tends to 1 as $s_w\to -\infty$, i.e. $v_0\to 0$.

In conclusion, our analysis gives a full picture of the critical behavior for cubic magnets that have $v_0>0$—we have $v_0=+\infty$ for fixed-length spins [42]. For $M=2$ the pure system has a critical XY transition for $s_y>-2/3$, an Ising transition for $s_y=-2/3$, and a first-order transition for $-1<s_y<-2/3$ [values of the parameters such that $s_y\leqslant -1$ are not allowed since they do not satisfy the stability bound (28)]. Note that first-order transitions cannot be observed in the pure model (15) with fixed-length spins. Indeed, for the strongest possible negative anisotropy, $K=-\infty$, the Hamiltonian can be written as two decoupled Ising models [43], and thus the system with $K=-\infty$ exactly corresponds to $s_y=-2/3$. As a consequence, finite values of K have $s_y>-2/3$, and therefore the model is expected to always have an XY transition. Randomness changes the critical behavior. For small randomness and small anisotropy, we always predict REIM critical behavior, while for large disorder (unless we consider fixed-length spins; cf. Sec. III D) we do not expect a continuous transition. Note that the behavior of systems with $-1<s_y<-2/3$ remains unclear since in this region we are not able to resum reliably the perturbative expansions. In particular, we cannot clarify if softening occurs. A Monte Carlo simulation [44] found that model (15) with $K=-\infty$ has a continuous transition for small disorder, in agreement with our results.

For $M=3$ we expect a continuous transition for $s_y\geqslant 0$ and a first-order one for $s_y<0$. If we add randomness to systems with $s_y>0$, the continuous transition survives but now belongs to the REIM universality class; for large disorder the transition may disappear. For $s_y < 0$ one may observe softening, i.e., the first-order transition may be changed into a continuous one by small disorder. Note that softening always occurs for infinite disorder, $D_0 = +\infty$, in the model (5), independently of the sign of y_0, under mild assumptions on the distribution $P(q)$; see Sec. III D.

ACKNOWLEDGMENT

P.C. acknowledges financial support from EPSRC Grant No. GR/R83712/01.

APPENDIX A: RENORMALIZATION-GROUP DIMENSIONS OF BILINEAR OPERATORS IN THE CUBIC-SYMMETRIC Φ^4 THEORY

In this appendix we compute the RG dimensions of bilinear operators in the cubic-symmetric theory

$$\mathcal{H}_c = \int d^d x \left\{ \frac{1}{2}[\partial_\mu \varphi(x)]^2 + \frac{1}{2} r \varphi(x)^2 + \frac{1}{4!} u_0 [\varphi(x)^2]^2 + \frac{1}{4!} v_0 \sum_{i=1}^{N} \varphi_i(x)^4 \right\}, \tag{A1}$$

where φ is an N-component field. The bilinear operators can be written in terms of tensors belonging to different irreducible representations of the cubic group:

$$E = \frac{1}{N} \sum_k \varphi_k^2, \tag{A2}$$

$$U_i = \varphi_i^2 - \frac{1}{N} \sum_k \varphi_k^2, \tag{A3}$$

$$T_{ij} = \varphi_i \varphi_j, \; i \neq j. \tag{A4}$$

The RG dimension of the energy operator E is $y_E = 1/\nu$, where ν is the correlation-length exponent. The RG dimensions of the operators U_i and T_{ij}, respectively y_U and y_T, in the cubic-symmetric theory (A1) will be computed below. Note that in O(N)-symmetric theories the tensors U_i and T_{ij} belong to the same irreducible representation and therefore $y_T = y_U$. In cubic systems this is no longer the case.

In order to compute y_U and y_T, we consider the perturbative approach in terms of the zero-momentum quartic couplings u and v at fixed dimension. We refer the reader to Ref. [27] for notation and definitions; there one can also find the six-loop perturbative expansion of the β functions and of the RG functions associated with the standard exponents. In order to compute the RG dimensions of U_i and T_{ij}, we consider the related RG functions Z_U and Z_T, defined in terms of the zero-momentum one-particle irreducible two-point functions $\Gamma_U^{(2)}(0)$ and $\Gamma_T^{(2)}(0)$ with the insertion of the operator U_i and T_{ij}, respectively, i.e.;

$$\Gamma_U^{(2)}(0)_{i,kl} = Z_U^{-1} B_{i,kl}, \; \Gamma_T^{(2)}(0)_{ij,kl} = Z_T^{-1} A_{ij,kl}, \tag{A5}$$

where B and A are appropriate constant tensors such that $Z_U = Z_T = 1$ at the tree level. Then we compute the RG functions η_U and η_T defined by

$$\eta_{U,T}(u,v) = \left. \frac{\partial \ln Z_{U,T}}{\partial \ln m} \right|_{u_0,v_0} = \beta_u \frac{\partial \ln Z_{U,T}}{\partial u} + \beta_v \frac{\partial \ln Z_{U,T}}{\partial v}, \tag{A6}$$

where β_u and β_v are the β functions.

We computed the functions $\Gamma_{U,T}^{(2)}(0)$ to six loops. The resulting six-loop series of $\eta_{U,T}(u,v)$ are

$$\eta_U(u,v) = -\frac{2u}{8+N} - \frac{1}{3} v + u^2 \frac{12+2N}{3(8+N)^2} + \frac{4}{3(N+8)} uv + \frac{2}{27} v^2 + \sum_{ij} e_{ij}^U u^i v^j, \tag{A7}$$

$$\eta_T(u,v) = -\frac{2u}{8+N} + u^2 \frac{12+2N}{3(8+N)^2} + \frac{4}{9(N+8)} uv + u \sum_{ij} e_{ij}^T u^i v^j, \tag{A8}$$

where u and v are normalized so that

$$mu = \frac{8+M}{48\pi} Z_u u_0, \; mv = \frac{3}{16\pi} Z_v v_0, \; Z_{u,v} = 1 + O(u,v), \tag{A9}$$

and the coefficients e_{ij}^U an e_{ij}^T are reported in Tables II and III, respectively. Note that u and v correspond to $\overline{u}$ and $\overline{v}$ in Ref. [27]. The RG dimensions y_U and y_T are obtained by $y_{U,T} = 2 + \eta_{U,T} - \eta$, where $\eta_{U,T}$ is the value obtained by resumming the corresponding series, evaluating it at $u = u^*$, $v = v^*$, where (u^*, v^*), is the stable FP.

In the case of the REIM, i.e., in the limit $N \to 0$, one has $y_U = y_E = 1/\nu$. Indeed, $\langle \varphi_i^2 \varphi_k \varphi_l \rangle^{1PI}$ and $\langle \Sigma_i \varphi_i^2 \varphi_k \varphi_l \rangle^{1PI}$ are both finite and nonvanishing for $N \to 0$. Therefore, we have for $N \to 0$

$$\langle N U_i \varphi_k \varphi_l \rangle^{1PI} = -\left\langle \sum_j \varphi_j^2 \varphi_k \varphi_l \right\rangle^{1PI} + O(N) = -\langle N E \varphi_k \varphi_l \rangle^{1PI} + O(N). \tag{A10}$$

Using the Monte Carlo estimate $\nu = 0.683(3)$ (Ref. [19]), we obtain $y_U = 1.464(6)$. The series for η_T was already reported in Ref. [45]. Its analysis provided the estimate $y_T = 2.08(3)$.

For $N = 2$ the stable FP of the cubic theory is the O(2) FP, so $y_U = y_T = 1.766(3)$ [26]. For $N \geqslant 3$ the O(N) FP is unstable and the RG trajectories flow toward another FP characterized by a discrete cubic symmetry. The analysis of the series, using the same procedure reported in Ref. [27], gives the estimates

$$y_U(N=3) = 1.774(7), \; y_T(N=3) = 1.800(2), \tag{A11}$$

$$y_U(N=4) = 1.696(8), \; y_T(N=4) = 1.874(3). \tag{A12}$$

TABLE II. The coefficients e_{ij}^U [cf. Eq. (A7)].

i,j	$(N+8)^i e_{ij}^U$
3,0	$-18.3128-3.43328\,N+0.216746\,N^2$
2,1	$-9.15642-0.17027\,N$
1,2	-1.17334
0,3	-0.0443103
4,0	$140.799+37.5734\,N+1.03627\,N^2+0.0943426\,N^3$
3,1	$93.8662+5.52597\,N-0.0781363\,N^2$
2,2	$18.4511-0.0667897\,N$
1,3	1.40677
0,4	0.0395196
5,0	$-1340.07-416.717\,N-17.6226\,N^2+0.911281\,N^3+0.0508337\,N^4$
4,1	$-1116.73-98.6844\,N+1.52723\,N^2-0.0301952\,N^3$
3,2	$-298.289-2.54766\,N-0.0492195\,N^2$
2,3	$-35.2065+0.140508\,N$
1,4	-1.98589
0,5	-0.0444004
6,0	$15651.3+5665.65\,N+433.687\,N^2+1.06755\,N^3+0.679106\,N^4+0.031393\,N^5$
5,1	$15651.3+1935.63\,N+8.74297\,N^2+0.581411\,N^3-0.00927903\,N^4$
4,2	$5294.38+134.776\,N-1.13059\,N^2-0.038664\,N^3$
3,3	$848.418-2.51108\,N+0.0323227\,N^2$
2,4	$72.4799-0.318348\,N$
1,5	3.24291
0,6	0.0603632

TABLE III. The coefficients e_{ij}^T [cf. Eq. (A8)].

i,j	$(N+8)^i e_{ij}^T$
2,0	$-18.3128-3.43328\,N+0.216746\,N^2$
1,1	$-3.09273+0.216746\,N$
0,2	-0.0337239
3,0	$140.799+37.5734\,N+1.03627\,N^2+0.0943426\,N^3$
2,1	$39.0459+1.53797\,N+0.12579\,N^2$
1,2	$2.66843+0.0769829\,N$
0,3	0.0716893
4,0	$-1340.07-416.717\,N-17.6226\,N^2+0.911281\,N^3+0.0508337\,N^4$
3,1	$-497.159-32.4255\,N+1.57919\,N^2+0.0847229\,N^3$
2,2	$-53.6464+0.443225\,N+0.062623\,N^2$
1,3	$-2.39176+0.0256721\,N$
0,4	-0.0421612
5,0	$15651.3+5665.65\,N+433.687\,N^2+1.06755\,N^3+0.679106\,N^4+0.031393\,N^5$
4,1	$7460.04+849.888\,N+0.972279\,N^2+1.37677\,N^3+0.062786\,N^4$
3,2	$1175.99+25.2714\,N+1.12305\,N^2+0.0567755\,N^3$
2,3	$88.2226+0.60426\,N+0.0297911\,N^2$
1,4	$3.53359+0.0136874\,N$
0,5	0.0607723

APPENDIX B: SOME RENORMALIZATION-GROUP IDENTITIES

In this appendix we prove relations (45). Morever, we show that, in the limit $N\to 0$, the RG functions do not depend on M for $v=0$.

Let us consider the Hamiltonian for $v_0=0$ and rewrite

$$\exp\left[-\frac{1}{4!}\sum_{ij,ab}(u_0+w_0\delta_{ab}+y_0\delta_{ab}\delta_{ij})\phi_{ai}^2\phi_{bj}^2\right]\sim\int d\lambda d\rho_a d\sigma_{ai}\times\exp\left[\frac{1}{2}\left(\lambda^2+\sum_a\rho_a^2+\sum_{ai}\sigma_{ai}^2\right)+\frac{1}{2\sqrt{3}}\sum_{ai}(\sqrt{u_0}\lambda+\sqrt{w_0}\rho_a+\sqrt{y_0}\sigma_{ai})\phi_{ai}^2\right], \tag{B1}$$

where $a(i)$ runs from 1 to $M(N)$ and λ, ρ_a, and σ_{ai} are auxiliary fields. Then let us consider the n-point irreducible correlation function. We will show that it has the form

$$\langle\phi_{a_1i_1}\cdots\phi_{a_ni_n}\rangle=\sum_\alpha c_\alpha Q_\alpha^{a_1i_1\cdots a_ni_n}, \tag{B2}$$

where Q_α are group tensors (products of Kronecker δ functions), the scalar factors c_α do not depend on M, and the sum runs over all possible independent group tensors. In terms of the auxiliary fields, Feynman diagrams contain loops of ϕ fields and $n/2$ open lines of ϕ fields connected by the auxiliary-field lines. The group factor associated with each diagram is computed as follows. One assigns indices ai to ϕ and σ propagators and indices a to ρ propagators, considers the product of the factors V (reported below) associated with each vertex, and sums over all assigned indices. In order to prove the M independence we will show that, because of the Kronecker δ's appearing in the diagrams, none of these sums is effectively performed in the limit $N\to 0$, so that no factor of M can appear. The factors V are given by

$$V(\lambda,\phi_{ai},\phi_{bj})=\frac{\sqrt{u_0}}{\sqrt{3}}\delta_{ab}\delta_{ij},$$

$$V(\rho_c,\phi_{ai},\phi_{bj})=\frac{\sqrt{w_0}}{\sqrt{3}}\delta_{abc}\delta_{ij},$$

$$V(\sigma_{ck},\phi_{ai},\phi_{bj})=\frac{\sqrt{y_0}}{\sqrt{3}}\delta_{abc}\delta_{ijk}, \tag{B3}$$

where $abc(ijk)$ run from 1 to $M(N)$. First, note that all ϕ loops must contain at least a $\sigma\phi\phi$ vertex, otherwise by summing over the indices of the ϕ fields appearing in the loop one obtains a factor of N. As a consequence, all loops give rise to a very simple effective vertex for the auxiliary fields:

$$\langle \lambda \cdots \lambda \rho_{a_1} \cdots \rho_{a_n} \sigma_{b_1 i_1} \cdots \sigma_{b_m i_m} \rangle \sim \delta_{a_1 \cdots a_n b_1 \cdots b_m} \delta_{i_1 \cdots i_m}, \tag{B4}$$

where we have written only the dependence on the group indices. Then, given a diagram, let us consider the reduced diagram in which all ϕ loops are replaced by the corresponding effective vertices. The σ_{ai} propagators form several connected paths. It is easy to convince oneself that each of these paths must end at an open ϕ line; otherwise, by summing over the indices i associated with the σ lines, one obtains factors of N. As a consequence, all effective vertices are connected by σ propagators to the open ϕ lines. Therefore, by summing over the indices associated with the ϕ and σ propagators one obtains expressions in which all remaining indices (those related to ρ propagators) are equal to external ones and are not summed over. We have thus proved that correlation functions expressed in terms of the bare parameters do not depend on M. Since R_{MN} is M independent for $N\rightarrow 0$, this result extends trivially to the RG functions expressed in terms of the renormalized couplings.

To prove the identities (45) we now exploit the M independence. For $M=1$ the theory corresponds to the REIM model with couplings u_0+w_0 and y_0. Since $R_{MN}=R_N$ for $N\rightarrow 0$, $(u+v)/m$ is a function of u_0+w_0. The result follows immediately.

[1] R. Harris, M. Plischke, and M. J. Zuckermann, Phys. Rev. Lett. **31**, 160 (1973).
[2] R. W. Cochrane, R. Harris, and M. J. Zuckermann, Phys. Rep. **48**, 1 (1978).
[3] P. M. Gehring, M. B. Salamon, A. del Moral, and J. I. Arnaudas, Phys. Rev. B **41**, 9134 (1990).
[4] A. del Moral, J. I. Arnaudas, P. M. Gehring, M. B. Salamon, C. Ritter, E. Joven, and J. Cullen, Phys. Rev. B **47**, 7892 (1993).
[5] Y. Imry and S.-k. Ma, Phys. Rev. Lett. **35**, 1399 (1975).
[6] R. A. Pelcovits, E. Pytte, and J. Rudnick, Phys. Rev. Lett. **40**, 476 (1978); **48**, 1297 (1982); S.-K. Ma and J. Rudnick, *ibid.* **40**, 589 (1978).
[7] A. Aharony and E. Pytte, Phys. Rev. Lett. **45**, 1583 (1980); Phys. Rev. B **27**, 5872 (1983).
[8] D. E. Feldman, Pis'ma Zh. Eksp. Teor. Fiz. **70**, 130 (1999) [JETP Lett. **70**, 135 (1999)]; Phys. Rev. B **61**, 382 (2000).
[9] D. E. Feldman, Int. J. Mod. Phys. B **15**, 2945 (2001).
[10] D. S. Fisher, Phys. Rev. B **31**, 7233 (1985).
[11] A. Aharony, Phys. Rev. B **12**, 1038 (1975).
[12] D. Mukamel and G. Grinstein, Phys. Rev. B **25**, 381 (1982).
[13] M. Dudka, R. Folk, and Yu. Holovatch, Condens. Matter Phys. **4**, 77 (2001).
[14] We have extended the perturbative series for the isotropic RAM to five loops in the fixed-dimension massive zero-momentum scheme. The analysis does not find evidence of stable fixed points.
[15] R. Fisch, Phys. Rev. B **58**, 5684 (1998).
[16] M. Itakura, Phys. Rev. B **68**, 100405(R) (2003).
[17] M. Dudka, R. Folk, and Yu. Holovatch, Condens. Matter Phys. **4**, 459 (2001); in *Fluctuating Paths and Fields*, edited by W. Janke, A. Pelster, H.-J. Schmidt, and M. Bachmann (World Scientific, Singapore, 2001) p. 457.
[18] Yu. Holovatch, V. Blavats'ka, M. Dudka, C. von Ferber, R. Folk, and T. Yavors'kii, Int. J. Mod. Phys. B **16** 4027 (2003).
[19] P. Calabrese, V. Martín-Mayor, A. Pelissetto, and E. Vicari, Phys. Rev. E **68**, 036136 (2003).
[20] H. G. Ballesteros, L. A. Fernández, V. Martín-Mayor, A. Muñoz Sudupe, G. Parisi, and J. J. Ruiz-Lorenzo, Phys. Rev. B **58**, 2740 (1998).
[21] P. Calabrese, P. Parruccini, A. Pelissetto, and E. Vicari, Phys. Rev. E **69**, 036120 (2004).
[22] P. Calabrese, A. Pelissetto, and E. Vicari, Phys. Rev. B **67**, 024418 (2003).
[23] A. Aharony, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic Press, New York, 1976), Vol. 6, p. 357.
[24] A. Pelissetto and E. Vicari, Phys. Rev. B **62**, 6393 (2000).
[25] A. Pelissetto and E. Vicari, Phys. Rep. **368**, 549 (2002).
[26] P. Calabrese, A. Pelissetto, and E. Vicari, Phys. Rev. B **67**, 054505 (2003); Phys. Rev. E **65**, 046115 (2002).
[27] J. M. Carmona, A. Pelissetto, and E. Vicari, Phys. Rev. B **61**, 15 136 (2000).
[28] P. Calabrese, A. Pelissetto, and E. Vicari, in *Frontiers in Superconductivity Research*, edited by Barry P. Martins (Nova Science, Hauppauge, NY, 2004); e-print cond-mat/0306273.
[29] H. G. Ballesteros, L. A. Fernández, V. Martín-Mayor, and A. Muñoz Sudupe, Phys. Lett. B **387**, 125 (1996).
[30] $DyAl_2$ is a ferromagnet with strong crystalline anisotropy, [100] being the easy axis for magnetization. Thus, we expect this system to show a first-order transition (see Ref. [25]). Experiments [3], however, apparently observe a Heisenberg second-order transition. This means that the first-order transition is very weak and that, in the range of temperatures investigated experimentally, $DyAl_2$ can be effectively considered as a Heisenberg system. It therefore makes sense to compare the experimental results for $(Dy_xY_{1-x})Al_2$ with the theoretical predictions for the crossover close to a Heisenberg transition.
[31] R. Folk, Yu. Holovatch, and T. Yavors'kii, Usp. Fiz. Nauk **173**, 175 (2003) Phys. Usp. **46**, 175 (2003)].
[32] P. Calabrese, M. De Prato, A. Pelissetto, and E. Vicari, Phys. Rev. B **68**, 134418 (2003).
[33] H. G. Ballesteros, L. A. Fernández, V. Martín-Mayor, A. Muñoz Sudupe, G. Parisi, and J. J. Ruiz-Lorenzo, J. Phys. A **32**, 1 (1999).
[34] B. G. Nickel, D. I. Meiron, and G. A. Baker, Jr., Guelph University Report, 1977 (unpublished).
[35] P. Calabrese, A. Pelissetto, and E. Vicari, e-print cond-mat/0311576.
[36] J. C. Le Guillou and J. Zinn-Justin, Phys. Rev. B **21**, 3976 (1980).
[37] A. J. Bray, T. McCarthy, M. A. Moore, J. D. Reger, and A. P. Young, Phys. Rev. B **36**, 2212 (1987); A. J. McKane, *ibid.* **49**, 12 003 (1994).
[38] In recent work [G. Álvarez, V. Martín-Mayor, and J. J. Ruiz-

Lorenzo, J. Phys. A **33**, 841 (2000)] a resummation method has been proposed that avoids, at least in the zero-dimensional REIM field theory, the problems noted in Ref. [37]. However, this method probably does not solve the problem in more than zero dimensions. Numerically, it apparently performs like the standard Padé-Borel or the direct conformal method (see Ref. [24] and the discussion in Refs. [21,19]).

[39] R. Guida and J. Zinn-Justin, J. Phys. A **31**, 8103 (1998).

[40] A. Pelissetto, and E. Vicari, Nucl. Phys. B **575**, 579 (2000); **519**, 626 (1998).

[41] Given an expansion $\Sigma c_n g^n$, we define the Borel-Leroy transform $B(t)=c_n t^n/\Gamma(n+b+1)$, and a Padé approximant $P(t)$ to $B(t)$. Then, a resummed expression is obtained by considering $\int_0^\infty dt t^b e^{-t} P(t)$. The parameter b may be chosen at will.

[42] Indeed, for $D=0$, the potential appearing in the Hamiltonian (20) can be written as $V(\varphi)=(1/4!)v_0(\vec{\varphi}^2+6r/v_0)^2+(1/4!)y_0\Sigma_a\varphi_a^4$. Fixed-length spins are recovered by taking the limit $v_0\to+\infty$, $r\to-\infty$ keeping $r/v_0=-1/6$.

[43] For $K\to-\infty$ the variables $\vec{s}$ are constrained to assume values $(\pm1,0)$ and $(0,\pm1)$. Thus, we can define $s_1=(1/2)(\sigma+\tau)$ and $s_2=(1/2)(\sigma-\tau)$, where σ and τ are unconstrained Ising variables. The Hamiltonian becomes that of two uncoupled Ising models, since $\vec{s}_x\cdot\vec{s}_y=(1/2)(\sigma_x\sigma_y+\tau_x\tau_y)$.

[44] R. Fisch, Phys. Rev. B **48**, 15 764 (1993).

[45] P. Calabrese, A. Pelissetto, and E. Vicari, Phys. Rev. B **68**, 092409 (2003).